Conjecture on Hidden Superconformal Symmetry of N=4 Supergravity

Sergio Ferrara[1,2], **Renata Kallosh**[3], and **Antoine Van Proeyen**[4]
[1] *Physics Department, Theory Unit, CERN CH 1211, Geneva 23, Switzerland*
[2] *INFN - Laboratori Nazionali di Frascati, Via Enrico Fermi 40, 00044 Frascati, Italy*
[3] *Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305*
[4] *Instituut voor Theoretische Fysica, KU Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium*

(Dated: September 4, 2012)

We argue that the observed UV finiteness of the 3-loop extended supergravities may be a manifestation of a hidden local superconformal symmetry of supergravity. We focus on the $SU(2,2|4)$ dimensionless superconformal model. In Poincaré gauge where the compensators are fixed to $\phi^2 = 6M_P^2$ this model becomes a pure classical N=4 Einstein supergravity. We argue that in N=4 the higher-derivative superconformal invariants like $\phi^{-4}W^2\bar{W}^2$ and the consistent local anomaly $\delta(\ln\phi\, W^2)$ are not available. This conjecture on hidden local N=4 superconformal symmetry of Poincaré supergravity may be supported by subsequent loop computations.

I. INTRODUCTION

The purpose of this note is to address the following issue: what if extended supergravity is perturbatively finite? Even if it is true (which of course we do not know at present) why could it be important? Is it possible that the conjectured perturbative UV finiteness may reveal some hidden symmetry of gravity? *Here we propose a conjecture that such a hidden symmetry may be an N=4 local superconformal symmetry.* If the 4-loop N=4 supergravity is UV divergent, this conjecture will be invalidated and, if it is UV finite, the conjecture will be supported.

Starting from the early days of supergravity, the superconformal calculus was a major tool for constructing new Poincaré supergravity models, see for example [1] and the recent book [2], which describes in detail the superconformal origin of N=1, 2 supergravities, including the role of the compensators and the gauge-fixing of the superconformal models down to super Poincaré. Extended N$\leq$ 4 supergravity models were developed in [3–6], starting with superconformal symmetry.

N> 4 supergravity models do not have an underlying superconformal symmetry. This is related to the fact that there are no matter multiplets, only pure supergravity multiplets are available. In particular, there is no supersymmetric extension of the square of the Weyl tensor in N> 4, as shown in [7].

Here we would like to suggest a possibility that the superconformally symmetric model underlying N=4 supergravity is not just a tool, but a major feature of a consistent perturbative supersymmetric theory involving gravity. Namely, we will show that the 3-loop finiteness of pure[1] N=4 supergravity [8], taken together with the absence of a candidate for a consistent local N=4 superconformal anomalies suggests that the principle of local superconformal symmetry may control the quantum properties of the gravitational theory, in the same way as the principle of non-abelian gauge symmetry controls the quantum properties of the standard model.

We are not used to think of a local 4-dimensional conformal symmetry as a reliable gauge symmetry, where the gauge-fixing and the ghosts structure support the BRST symmetry and the computations confirm the formal properties of the path integral. The common expectation is that this symmetry may be unreliable because of anomalies. Therefore we cannot use it for investigation of divergences in the usual Einstein gravity.

The N=4 local superconformal symmetry may be an example of an anomaly-free theory, and therefore it is tempting to study possible implications of the local superconformal symmetry, starting with this case. In particular, the absence of the 3-loop UV divergences in pure N=4 supergravity [8] may be interpreted as a manifestation of the superconformal symmetry of the un-gauge-fixed version of this theory.

We will discuss possible implications of the conjecture of hidden N=4 superconformal symmetry for the all-loop UV properties of N$\geq$ 4 supergravities [2]. In N> 4, in ab-

[1] Our analysis is not valid for the case of N=4 supergravity interacting with matter, studied in [9]. These models have a 1-loop UV divergence [10].

[2] Some early hints about the possibility of UV finiteness of N$\geq$ 5

sence of duality anomalies [11], the duality current conservation argument can be used towards the UV finiteness of the perturbative supergravity [13]. In N=4 case, where there is a 1-loop global $U(1)$ duality anomaly [11], one might have some concerns regarding the explanation of the 3-loop UV finiteness in pure N=4 supergravity[12]. However, we will argue below that in the underlying superconformal N=4 model the local superconformal symmetry is anomaly free.

II. CONFORMAL COMPENSATOR IN N=0 SUPERGRAVITY

Consider a model of pure gravity, N=0, promoted to a local Weyl conformal symmetry:

$$S^{conf} = \frac{1}{2}\int d^4x\sqrt{-g}\left(\partial_\mu\phi\,\partial_\nu\phi\,g^{\mu\nu} + \frac{1}{6}\phi^2\,R\right). \quad (1)$$

The field ϕ is referred to as a conformal compensator. Various aspects of this toy model of gravity with a Weyl compensator field (1) were studied over the years [15]. The action is conformal invariant under the following local Weyl transformations:

$$g'_{\mu\nu} = e^{-2\sigma(x)}g_{\mu\nu}\,, \qquad \phi' = e^{\sigma(x)}\phi\,. \quad (2)$$

The gauge symmetry (2) with one local gauge parameter can be gauge fixed. We may choose the unitary gauge

$$\phi^2 = \frac{6}{\kappa^2} \quad (3)$$

Note that one has to take a scalar field with ghost-like sign for the kinetic term to obtain the right kinetic term for the graviton. This does not lead to any problems since this field disappears after the gauge fixing and the action (1) reduces to the Einstein action, which is not conformally invariant anymore:

$$S^{conf}_{gauge-fixed} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,R\,. \quad (4)$$

In this action, the transformation (2) does not leave the Einstein action invariant any more. The R-term transforms with derivatives of $\sigma(x)$, which in the action (1) were compensated by the kinetic term of the compensator field and the weight was compensated by ϕ^2 term which is not present in the gauge-fixed action anymore. But the general covariance is still the remaining local symmetry of the action.

and not only N=8 supergravity were given in [14] based on the observation that generic theories of quantum gravity based on the Einstein-Hilbert action may be better behaved in UV at higher loops than suggested by naive power counting.

Now let us look for the consequences of our conjecture that the local (super)conformal symmetry is fundamental, instead of Poincaré (super)gravity.

1. We know that the first UV divergence that was predicted in pure gravity (not taking into account the conformal predictions, but only general covariance) at the 2-loop level [16] is given by the cube of the Weyl tensor

$$\Gamma_2^{N=0} \sim \frac{1}{\epsilon}\kappa^2\int d^4x\sqrt{-g}\,C_{\mu\nu}{}^{\lambda\delta}C_{\lambda\delta}{}^{\alpha\beta}C_{\alpha\beta}{}^{\mu\nu}\,. \quad (5)$$

Here the on-shell condition is $R = R_{\mu\nu} = 0$ and $C_{\mu\nu\lambda\delta} = R_{\mu\nu\lambda\delta}$.

2. The actual computation was performed in [17], which demonstrated that 2-loop gravity is indeed UV divergent:

$$\Gamma_2^{N=0} = \frac{\kappa^2}{(4\pi)^4}\frac{209}{2880}\frac{1}{\epsilon}\int d^4x\sqrt{-g}\,C_{\mu\nu}{}^{\lambda\delta}C_{\lambda\delta}{}^{\alpha\beta}C_{\alpha\beta}{}^{\mu\nu}\,. \quad (6)$$

This finalized a convincing story of the UV infinities in pure N=0 quantum gravity. There is no reason to expect that the 3-loop counterterm as well as all higher loop order $\frac{1}{\epsilon}$ UV divergences, will not show up.

Now assume that we use the underlying conformal model with local conformal symmetry. It is easy to promote the 2-loop UV divergence to the form of a conformal invariant:

$$\int d^4x\sqrt{-g}\,\phi^{-2}\,C_{\mu\nu}{}^{\lambda\delta}C_{\lambda\delta}{}^{\alpha\beta}C_{\alpha\beta}{}^{\mu\nu}\,. \quad (7)$$

Upon gauge-fixing it will produce the candidate for the 2-loop divergence. Thus, even if we would use the embedding of gravity into a model with conformal symmetry by introducing an extra scalar- compensator, it would not help us to forbid the 2-loop UV divergence in the N=0 supergravity.

A. N=0 supergravity with matter

If we would add some additional matter to our superconformal N=0 toy model, we would have to consider the 1-loop conformal counterterm independent on the compensator field, proportional to the square of the Weyl tensor

$$\Gamma_1^{N=0} \sim \frac{1}{\epsilon}\int d^4x\sqrt{-g}\,C_{\mu\nu\lambda\delta}C^{\lambda\delta\mu\nu}\,. \quad (8)$$

and in the topologically trivial background this counterterm is

$$\Gamma_1^{N=0} \sim \frac{2}{\epsilon}\int d^4x\sqrt{-g}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)\,. \quad (9)$$

The coefficient in front depends on the matter content. The reason for its absence in pure gravity was explained using the background field method in [18] by the fact that it is proportional to classical equations of motion when the rhs of Einstein equation, $T^{mat}_{\mu\nu}$ is vanishing. This means that the relevant divergence can be removed by change of variables. In [19] it was shown explicitly that in pure gravity there is a choice of the parameters a, b of the general covariance gauge-fixing condition, of the type $aD_\mu h^{\mu\nu} - bD_\mu h = 0$ which makes both the UV divergences $R_{\mu\nu}R^{\mu\nu}$ and R^2 a, b-gauge-dependent, and vanishing at certain values of a, b.

Thus, in the exceptional case of pure gravity without matter there is a 1-loop UV finiteness, the same is valid for all pure supergravities without matter. However, in presence of matter in gravity as well as in supergravities, the 1-loop UV divergences that are present are defined by the matter part of the energy momentum tensor.

$$\Gamma_1^{N=0} = \frac{1}{\epsilon}\int d^4x\sqrt{-g}\left(\alpha(T^{mat}_{\mu\nu})^2 + \beta(T^{mat})^2\right) . \quad (10)$$

where α and β depend on the matter content of the given model.

III. N=1, N=2, N=4 SUPERGRAVITY

A. N=1,2

The generic N=1,2 supergravity models were derived by gauge-fixing the N=1,2 superconformal algebra, starting with $SU(2,2|1), SU(2,2|2)$, respectively, see [2] and references therein. The known facts are:

1. In N=1,2 supergravities the prediction was made in [20] that the 3-loop divergence of the form

$$\Gamma_3^{N=1,2} \sim \frac{1}{\epsilon}\kappa^4 \int d^4x\,\sqrt{-g}\,C_{\alpha\beta\gamma\delta}C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}C^{\alpha\beta\gamma\delta}C^{\dot\alpha\dot\beta\dot\gamma\dot\delta} \quad (11)$$

is possible.

2. There were no computations of the 3-loop UV divergence in N=1, N=2 supergravity so far.

The prediction in (11) was based on local supersymmetry, associated with Poincaré N=1,2 supergravity.

The superconformal embedding prediction would require us to provide the superconformal embedding of the term in (11). The question is: is there a N=1,2 superconformal generalization of the expression

$$\int d^4x\,\sqrt{-g}\,\phi^{-4}C_{\alpha\beta\gamma\delta}C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}C^{\alpha\beta\gamma\delta}C^{\dot\alpha\dot\beta\dot\gamma\dot\delta} , \quad (12)$$

which is the gravity part of the full N=1,2 superconformal higher derivative invariant. The answer is positive and is based on the fact that in N=2 there is a local superconformal calculus and there are chiral multiplets with arbitrary Weyl weight [21], in particular the negative powers of the compensator multiplet, that can be used for building higher derivative superconformal invariants. Moreover, various examples of superconformal higher derivative invariants in the N=2 model are presented in [22] and recently used for comparison with on shell superspace counterterms in [23]. The simplest N=2 superconformal version of R^4 corresponding to minimal pure N=2 supergravity is given by the following chiral superspace integral [23]

$$\lambda\int d^4\theta\left(\frac{W^2}{S^2}\mathbb{T}\left(\frac{\overline{W^2}}{\overline{S^2}}\right)\right) . \quad (13)$$

The N=2 superconformal calculus allows to use the chiral multiplets S^{-2} as well as any higher negative power S^{-2n} for building higher and higher derivative invariants in the N=2 supergravity [22]. Thus the hidden local superconformal N=2 symmetry does not lead to a particular restriction on N=2 supergravity counterterms.

B. N=4 supergravity

1. The prediction was made in [24] that the 3-loop divergence of the form

$$\Gamma_3^{N=4} \sim \frac{1}{\epsilon}\kappa^4 \int d^4x\,\sqrt{-g}\,C_{\alpha\beta\gamma\delta}C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}C^{\alpha\beta\gamma\delta}C^{\dot\alpha\dot\beta\dot\gamma\dot\delta} \quad (14)$$

is expected since the relevant candidate counterterm has all required non-linear symmetries of N=4 supergravity, including the $SU(1,1)\times SO(6)$ duality.

2. The recent computation in [8] revealed that

$$\Gamma_3^{N=4} = 0 . \quad (15)$$

The computations of UV loop divergences in [25] and in [8] are based on the information about the tree amplitudes and on the unitarity method. Therefore these computations seem to shed some light on all version of extended supergravity, which at the tree level are equivalent. Such versions are related by various classical duality transformations. We proceed from here by suggesting a conjecture of a hidden superconformal symmetry, which these computations may have revealed.

We will now proceed with the analysis based on our conjecture that the local superconformal supersymmetry may control the UV divergences of N=4 Poincaré supergravity.

IV. N=4 SUPERCONFORMAL SYMMETRY AND SUPERGRAVITY

Here we follow [3, 4] and specifically [5], where the details on N=4 case have been worked out. To derive N=4 supergravity from the superconformal model based on the $SU(2,2|4)$ graded algebra requires a number of rather complicated steps. We will only describe here the ones that are relevant for our purpose, referring the reader to the original papers [3–5].

To derive the action of a pure N=4 Poincaré supergravity one has to start with 6 (wrong sign) metric N=4 vector multiplets interacting with the N=4 Weyl gravitational multiplet. The Abelian vector multiplet action with the correct sign of the metric invariant under rigid N=4 supersymmetry is

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar\psi^i\gamma\cdot\partial\psi_i - \frac{1}{2}\partial_\mu\phi_{ij}\partial^\mu\phi^{ij}\ , \tag{16}$$

with $i,j = 1,...,4$. For the 6 compensator vector multiplets $(I,J = 1,\ldots,6)$ we take

$$-\frac{1}{4}F^I_{\mu\nu}\eta_{IJ}F^J_{\mu\nu} - \bar\psi^{iI}\eta_{IJ}\gamma\partial\psi^J_i - \frac{1}{2}\partial_\mu\phi^I_{ij}\eta_{IJ}\partial_\mu\phi^{ijJ}\ , \tag{17}$$

where $\phi^{ij} = (\phi_{ij})^* = \varepsilon^{ijk\ell}\phi_{k\ell}$ and the constant real metric η_{IJ} is diagonal and has 6 negative eigenvalues, -1. This action is invariant under global $SU(4)$. The six negative eigenvalues point towards the role of ϕ^I_{ij} as compensators of a conformal symmetry, as explained in the toy model above. In case of pure N=4 supergravity without matter multiplets all scalars from the 6 N=4 superconformal vector multiplets are the compensator scalars, as we will see below.

To derive the N=4 pure supergravity action one starts with 6 such vector multiplets and couple them to the fields of conformal N=4 supergravity. There is a derivative $\mathcal{D}_a = e^\mu_a\mathcal{D}_\mu$, which is covariant with respect to all superconformal symmetries of $SU(2,2|4)$. Meanwhile $\mathcal{D}_\mu$ is covariant under Lorentz, Weyl, $SU(4)$ and $U(1)$ symmetries. The S- and K-covariantization is performed in [5] explicitly.

The rigid supersymmetry algebra $\{Q,Q\}$ leads to translation P, so it is necessary to convert it into general coordinate transformations to describe the coupling with gravity. This and analogous steps require some constraints on the curvatures as well as introduction of fields, in addition to gauge fields above, to close the algebra, so that, after all

$$[\delta_Q(\epsilon_1),\delta_Q(\epsilon_2)] = \delta_G^{\rm cov}(\xi^\mu) + \delta_M(\epsilon^{ab}) + \delta_Q(\epsilon^i_3) + \delta_S(\eta^i) +$$

$$\delta_{SU(4)}(\lambda^i{}_j) + \delta_{U(1)}(\lambda_T) + \delta_K(\lambda^a_K) + \delta_A(\lambda) + X_{EOM} \tag{18}$$

The rhs of this commutator depends on a combination of all local symmetries: general covariant, Lorentz, supersymmetry, special supersymmetry, $SU(4)$, $U(1)$, conformal boosts, Abelian gauge transformation on the vector fields and spinor equation of motion on ψ^i, which we show in the last term in X_{EOM}. The explicit expressions are derived in [5]. The parameters of all these transformations, which form an ‘open algebra’, are bilinear in $\epsilon_1(x)$, $\epsilon_2(x)$. The constraints on the curvatures lead to certain relations between the gauge fields so that some of them are not independent anymore.

A. Superconformal coupling of vector multiplets to the Weyl multiplet

The N=4 superconformal Lagrangian of the vector multiplets interacting with the Weyl multiplet is given in eq. (3.16) in [5] and takes a full page. We will present here the bosonic part of the action for the 6 compensating vector multiplets, with the wrong sign of kinetic terms, which is relatively simple.

$$\begin{aligned} e^{-1}L^{bos}_{s.c.} = &-\frac{1}{4}F^{+I}_{\mu\nu}\eta_{IJ}F^{+J}_{\mu\nu}\frac{\phi^1-\phi^2}{\Phi} - \frac{1}{4}\mathcal{D}_\mu\phi^I_{ij}\eta_{IJ}\mathcal{D}_\mu\phi^{ijJ} \\ &- F^{+I}_{\mu\nu}\eta_{IJ}T^{\mu\nu}_{ij}\phi^{ijJ}\frac{1}{\Phi} - \frac{1}{2}T_{\mu\nu ij}\phi^{ijJ}\eta_{IJ}T^{\mu\nu}_{kl}\phi^{klJ}\frac{\Phi^*}{\Phi} \\ &- \frac{1}{48}\phi^I_{ij}\eta_{IJ}\phi^{ijJ}\left(E^{kl}E_{kl} + 4D_a\phi^\alpha D^a\phi_\alpha - 12f_\mu{}^\mu\right) \\ &+ \frac{1}{8}\phi^I_{ij}\eta_{IJ}\phi^{klJ}D^{ij}{}_{kl} + h.c. \end{aligned} \tag{19}$$

Here

$$\Phi = \phi^1 + \phi^2\ , \qquad \Phi^* = \phi_1 - \phi_2\ , \qquad \phi^\alpha\phi_\alpha = 1\ . \tag{20}$$

The conformal boost gauge field $f_\mu{}^a$ is a function of a curvature

$$f_\mu{}^\mu = -\frac{1}{6}R(\omega)\ . \tag{21}$$

The scalars ϕ_α with $\alpha = 1,2$ transform as a doublet under $SU(1,1)$. The constraint (20) and the $U(1)$ gauge invariance reduce the 2 complex variables ϕ_α to 2 real fields, so that they are in $\frac{SU(1,1)}{U(1)}$ coset space.

The fields $T_{\mu\nu ij}, E^{kl}, D^{ij}{}_{kl}$ belong to the gravitational Weyl multiplet. This action, supplemented by all fermionic terms, leads to equations of motion of fermion partner of the compensator scalars ψ^{iI} in the rhs of the commutator of two supersymmetries (18), which we denoted by X_{EOM}.

The action is linear in $D^{ij}{}_{kl}$, so it is convenient for the purpose of future gauge-fixing to use the reparametrization of the 36 variables $\phi_{ij}{}^I$ in terms of the 36 $\varphi_M{}^I(x) \equiv$

$\{\varphi^I_m(x), \varphi^I_{m+3}(x)\}$, so that $M = 1, ..., 6$:

$$\phi^I_{ij}(x) = \varphi^I_m(x)\beta^m{}_{ij} + \mathrm{i}\varphi^I_{m+3}(x)\alpha^m{}_{ij} \ , \qquad (22)$$

where α^m and β^m with $m = 1, 2, 3$ are $SU(2) \times SU(2)$ numerical matrices introduced in [26].

V. POINCARÉ GAUGE

The superconformal action has unbroken local K-, D- and S-symmetries which are not present in supergravity and must be gauge-fixed to convert the superconformal action into a supergravity one.[3] This is done the same way as in the toy example above, namely, the scalar compensator dependent term in front of R is designed to introduce a Planck mass into conformal theory which originally, before gauge-fixing, has no dimensionful parameters. To fix the local dilatation D one can take

$$\phi^I_{ij}\eta_{IJ}\phi^{ijJ} = -\frac{6}{\kappa^2} \ . \qquad (23)$$

This provides the Einstein curvature term in the action

$$-\frac{1}{24}\phi^I_{ij}\eta_{IJ}\phi^{ijJ}R \quad \Rightarrow \quad \frac{1}{4\kappa^2}R \qquad (24)$$

and explains why the diagonal metric η_{IJ} has six negative values. The S- and K- local symmetries are fixed by taking

$$b_\mu = 0 \ , \qquad \psi_i{}^J = 0 \ . \qquad (25)$$

The fact that our 6 vector multiplets have a wrong sign kinetic terms is in agreement with the fact that the scalars are conformal compensators. As long as $\varphi^I_m(x)$ and $\varphi^I_{m+3}(x)$ with $m = 1, 2, 3$ are present, there is also a local $SU(4)$ symmetry. So, we can use the 15 parameters from $SU(4)$ together with $20 + 1$ conditions: field equations of $D^{ij}{}_{k\ell}$ and the dilatation gauge mentioned in (27), to take the scalars in eq. (22) constant,

$$\varphi_M{}^I(x) = \frac{1}{2\kappa}\delta_M{}^I \ , \qquad (26)$$

to remove these 36 variables.

The remaining important steps include the elimination of the auxiliary fields of the Weyl multiplet, E^{kl} and $T_{\mu\nu ij}$. The field E^{kl} turned out to be proportional to fermion bilinears, which changes the fermionic part of the action. However, the role of $T_{\mu\nu ij}$ is extremely important: the procedure of its exclusion on its equations of motion leads to a sign conversion of the kinetic term for the vectors from the 6 vector multiplets, they become physical vectors with the correct sign kinetic term.

To summarize, the 6 vector multiplets at the superconformal stage all have wrong kinetic terms since the N=4 scalar partners play the role of conformal compensators. When scalars are gauge-fixed to eliminate the local Weyl D-symmetry (dilatation), the Einstein gravity arises. The 6 quartets of spinors ($6 \times 4 \times 4 = 96$ components) from the vector gauge multiplets are eliminated by the combination of 16 gauge conditions of local S-supersymmetry (special supersymmetry) and the field equations of the auxiliary fermions in the Weyl multiplet (80 components).

The vectors from the 6 vector multiplets are converted into physical vectors of supergravity, when the auxiliary field $T_{\mu\nu ij}$ of the Weyl multiplet is excluded on its equations of motion. The action becomes that of pure N=4 supergravity in $\kappa^2 = 1$ units where the local $U(1)$ symmetry is still present. The bosonic part is

$$\begin{aligned} e^{-1}L^{\text{bos}}_{\text{sg}} &= \frac{1}{4}R(\omega) + \frac{1}{2}D_a\phi^\alpha D^a\phi_\alpha + \\ &+ \frac{1}{4}F^{+I}_{\mu\nu}\eta_{IJ}F^{+J\mu\nu}\frac{\phi_1+\phi_2}{\phi_1-\phi_2} + h.c. \end{aligned} \qquad (27)$$

Note that the actions are supersymmetric after adding the fermionic part. This means that the variation of the action vanishes for arbitrary field configurations: the fields do not satisfy any equations. The statement that the multiplets are 'on shell' is a statement on the algebra of transformations, and that one depends on specific field equations. Thus no other invariant can be constructed with these on shell multiplets, since this would change the field equations. This is why the inverse powers of the vector multiplet (or a logarithmic function) can't be used to construct other invariant actions, see a further discussion of this in Sec. VI.

Triangular $U(1)$ gauge-fixing

The local $U(1)$ gauge we take[4] is

$$\mathrm{Im}\,(\phi_1 - \phi_2) = 0 \qquad (28)$$

Our choice is motivated by the triangular decomposition of the $SL(2,\mathbb{R})$ matrix of our model. We start with the $SU(1,1)$ matrix defined in [5]

$$U = \begin{pmatrix} \phi_1 & \phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} \qquad (29)$$

[3] This is a precise analog of 3 gauge symmetries in the $SU(3) \times SU(2) \times U(1)$ model which have been gauge-fixed in the unitary gauge where $W^\pm$ and Z are massive vector mesons.

[4] The related construction is discussed in [6], without making explicit choice of the $U(1)$ gauge.

We switch to the $SL(2,\mathbb{R})$ basis and get

$$S = \mathcal{A}\, U\, \mathcal{A}^{-1} = \begin{pmatrix} \mathrm{Re}(\phi_1+\phi_2) & -\mathrm{Im}(\phi_1+\phi_2) \\ \mathrm{Im}(\phi_1-\phi_2) & \mathrm{Re}(\phi_1-\phi_2) \end{pmatrix} \quad (30)$$

where

$$\mathcal{A} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \quad (31)$$

We define an independent variable τ as

$$\tau = \tau_1 + i\tau_2 \equiv i\frac{\phi_1+\phi_2}{\phi_1-\phi_2} \quad (32)$$

which parametrizes the coset space $\frac{SL(2,\mathbb{R})}{U(1)}$. We take

$$\phi_1 = \frac{1}{2\sqrt{\tau_2}}(1-i\tau) \qquad \phi_2 = -\frac{1}{2\sqrt{\tau_2}}(1+i\tau) \quad (33)$$

In this notation with $\tau_2 = e^{-2\varphi}$ and $\tau_1 = \chi$ the triangular decomposition of the $SL(2,\mathbb{R})$ matrix is clear

$$S = \begin{pmatrix} e^{-\varphi} & \chi e^{\varphi} \\ 0 & e^{+\varphi} \end{pmatrix} \quad (34)$$

When these values of ϕ_α are inserted into the superconformal action (3.16) or the partially gauge-fixed (4.18) of [5], we get the CSF N=4 supergravity model [26] the bosonic part of which is

$$\begin{aligned} e^{-1}L_{CSF} &= \frac{1}{2}R - \frac{1}{4}\frac{\partial\tau\partial\bar{\tau}}{(\mathrm{Im}\,\tau)^2} \\ &+ \frac{1}{4}\delta_{IJ}\left[i\tau F^{+I}_{\mu\nu}F^{+J\mu\nu} + h.c.\right] \end{aligned} \quad (35)$$

Bergshoeff, de Roo, de Wit $U(1)$ gauge

The choice in [4, 5] is

$$\mathrm{Im}\ \phi_1 = 0 \quad (36)$$

and the independent variable is defined as

$$Z \equiv \frac{\phi_2}{\phi_1} \quad (37)$$

Here the scalars

$$\phi_1 = \frac{1}{\sqrt{1-|Z|^2}}\ , \qquad \phi_2 = \frac{Z}{\sqrt{1-|Z|^2}}\ , \quad (38)$$

parametrize the coset space $\frac{SU(1,1)}{U(1)}$. The 6 vector multiplets have a correct sign kinetic terms. The theory has a global duality symmetry $SU(1,1)\times SO(6)$ inherited from the superconformal N=4 model. The scalar couplings are

$$-\frac{\partial Z\partial\bar{Z}}{(1-|Z|^2)^2} \quad (39)$$

In this local $U(1)$ gauge the N=4 supergravity is an intermediate version between the CSF model [26] and the one given in [27] and in full details in [28]. Specifically, the scalars Z are the same as in [28], however, the vectors are related to the ones in [28] by a duality transformation.

Thus we find that our new gauge which provides a CSF N=4 supergravity model [26] directly from the superconformal model is nice and simple, comparative to other versions of N=4 supergravity.

VI. HIGHER DERIVATIVE SUPERCONFORMAL ACTIONS IN N=4 MODEL

There is only one type of possible matter multiplets in N=4 supersymmetry, i.e. N=4 Maxwell multiplets (and the non-abelian version). The scalar ϕ_{ij} has Weyl weight $w=1$ and therefore it is used to gauge-fix local dilatation by the $\phi^I_{ij}\eta_{IJ}\phi^{ijJ} = -\frac{6}{\kappa^2}$ condition. The fact that the algebra does not close on these N=4 Maxwell multiplets implies that we cannot use them anymore in further tensor calculus for N=4. Moreover, the local superconformal symmetry algebra is closed on the Weyl multiplet. Since there are no other multiplets in that case, it is not possible to construct the N=4 superconformal version of C^4 shown in eq. (12), which requires a superfield with the conformal weight $w=-4$ which in the Poincaré gauge becomes κ^4. It would require an N=4 superconformal version of the bosonic expression $(\phi^I_{ij}\eta_{IJ}\phi^{ijJ})^{-2}C^4$, which does not exist. Including supercovariant derivatives $D_a = e^\mu_a D_\mu$ with $w(e^\mu_a) = +1$ can only increase the positive conformal weight w of the corresponding superconformal invariant, which requires higher negative powers of a compensator.

The situation in the N=4 case is in sharp contrast with N=1,2 cases where there are chiral superfields of arbitrary conformal weight w [21], which can be used to build the superconformal invariants. Moreover, according to eq. (C.2) in [22] one can take an arbitrary function of the chiral compensator superfield $\mathcal{G}(\phi)$ and construct such a negative conformal weight superfield out of the compensators in the N=2 superconformal case. In Poincaré gauge such a superfield $\mathcal{G}(\phi) = \phi^{-2n}$ will provide the increasing powers of gravitational coupling $\phi^{-2n} \Rightarrow \kappa^{2n}$.

To explain why in N=4 superconformal theory it is not possible to produce superinvariant actions with arbitrary function of superfields consider an example : the off-shell chiral multiplet

$$(z, \chi_L, F) \quad (40)$$

We want to construct an action $S = \int d^2\theta\mathcal{G}(z)$. To find the components, we obtain the fermion component of $\mathcal{G}$ by calculating one susy transformation on the lowest component $\mathcal{G}(z)$. This gives $\mathcal{G}'(z)\chi_L$. A further transfor-

mation gives (for the component that will be integrated)

$$\mathcal{G}'(z)F - (1/2)\mathcal{G}''(z)\bar{\chi}_L\chi_L \tag{41}$$

This transforms to $\gamma^\mu\partial_\mu[\mathcal{G}'(z)\chi_L]$ and thus gives a good invariant action.

However, consider now that we would have only the on-shell multiplet, e.g. for a massless multiplet. Then $F = 0$. The algebra on χ_L leads then to the field equation $\gamma^\mu\partial_\mu\chi_L = 0$. The second susy transformation as above leads to $-(1/2)\mathcal{G}''(z)\bar{\chi}_L\chi_L$ Thus the superfield would be

$$\mathcal{G}(z) + \bar{\theta}_L\mathcal{G}'(z)\chi_L - (1/2)\mathcal{G}''(z)\bar{\theta}_L\theta_L\bar{\chi}_L\chi_L \tag{42}$$

This is a superfield for any $\mathcal{G}(z)$. However, the integral $\int d^2\theta$ gives the last component, which transforms under susy to

$$\gamma^\mu\chi_L\partial_\mu\mathcal{G}'(z) \tag{43}$$

This is not a total derivative (missing a term proportional to the field equation, but that we cannot use to have an invariant action). This illustrates that the multiplet calculus can only be used for off-shell multiplets. We provide a more detailed discussion and relation to N=2 deformation in models with higher derivatives in the Appendix.

Before we take seriously a prediction on higher derivative superinvariants following from the local N=4 superconformal theory, we have to study the situation with anomalies. The local anomalies for N=1 superconformal theories were studied in [29] and in [30, 31]. Our N=4 superconformal model of 6 (wrong sign) compensators interacting with the Weyl multiplet, upon gauge-fixing, leads to pure N=4 supergravity with Einstein curvature, without the square of the curvature in the action. The local anomalies of this model will be discussed below.

A. Superconformal anomalies

Local superconformal anomalies were studied in detail in N=1 case in [29]. It was explained there that the consistent anomalies can be constructed using the Wess-Zumino method [32]. In gauge theories the method allows to construct terms $\Gamma(\Phi, A_\mu)$, whose variation takes a form of a consistent anomaly $\delta_\Lambda\Gamma(\Phi, A_\mu)$, which does not depend on the compensator field Φ. Later the related work was performed in a somewhat different context in [30] based on [31]. We are interested in local symmetry anomalies, which in gauge theory examples may be fatal and lead to a quantum inconsistent theory. For example, the triangle local chiral symmetry anomaly in standard model, if not compensated, means that the physical observables in the unitary gauge do not coincide with the physical observables in the renormalizable gauge. The change of variables in the path integral of the kind performed in [33], which allows to prove an equivalence theorem for the S-matrix in arbitrary gauges, may be invalidated in presence of anomalies.

For example, in the simple case of $SU(2)$ gauge model we may be interested in transverse renormalizable gauge $\partial^\mu A_\mu{}^m = 0$ or in the unitary gauge $B^m = 0$. To find the relation between these two gauges one may look at a more general class of gauges like $a\,\partial^\mu A_\mu{}^m + b\,B^m = 0$. In the unitary gauge the theory is not renormalizable off shell, however, if the equivalence theorem

$$\langle|S|\rangle|_{a,b} = \langle|S|\rangle|_{a+\delta a, b+\delta b} \tag{44}$$

is valid, the physical observable are the same as the ones in renormalizable gauge (with account of some dependence on gauge-fixing of renormalization procedure). Also the proof of unitarity in the renormalizable gauge is based on validity of (44).

Local symmetry anomaly may invalidate the a, b independence of physical observables. Instead of equivalence we have a relation

$$\langle|S|\rangle|_{a,b} = \langle|S|\rangle|_{a+\delta a, b+\delta b} + X\langle\int\Lambda^\alpha(x, \phi^i, \delta a, \delta b)\,\mathcal{A}_\alpha(\phi^i)\rangle \tag{45}$$

Here $\mathcal{A}_\alpha(\phi^i)$ is the consistent anomaly depending on various fields ϕ^i of the model, and $\Lambda^\alpha(x, \phi^i, \delta a, \delta b)$ is a specific change of variables, leaving the classical action invariant, but effectively changing the gauge-fixing condition, with examples given in [33]. X is a numerical value in front of a candidate anomaly, which may vanish, in case of cancellation, or not, depending on the model.

Thus, in the context of local anomalies which may exist and destroy the quantum consistency of the model, we will look at possible candidates for anomalies given by expressions like

$$\delta_\Lambda\Gamma(\phi^i) = \int d^4x\,\Lambda^\alpha(x)\,\mathcal{A}_\alpha(\phi^i) \tag{46}$$

where $\Lambda^\alpha(x)$ corresponds to all gauge symmetries of a given model.

There are 2 conditions for an anomaly to be fatal for a gauge theory, i. e. to make quantum theory inconsistent.

I. The candidate consistent anomaly (46) should be available according to local symmetries of the model

II. The numerical coefficient in front of a candidate anomaly, which is due to contribution from various fields of the model, should not cancel, $X \neq 0$ in (45).

N=1 case

The symmetries of N=1 superconformal models in-

clude

$$\epsilon(x)\,, \qquad \eta(x)\,, \qquad \lambda_D(x)\,, \qquad \lambda_T(x) \tag{47}$$

i.e. local Q-supersymmetry, local S-supersymmetry, Weyl local conformal symmetry, local chiral U(1) symmetry, respectively, and of course, general covariance and Lorentz symmetry.

In case of N=1 superconformal models the corresponding $\delta_\Lambda \Gamma(\phi, W^2)$ was given in [29] in eq. (5.7). The integrated form of the anomaly is given by a local action in eq. (5.6) in [29]

$$\Gamma^{dWG}(\phi, W^2) \tag{48}$$

Here ϕ is the compensator superfield of a Weyl weight $w = 1$, and $W_{\alpha\beta\gamma}$ is a Weyl superfield of conformal weight $w = 3/2$. The variation of (48) produces a consistent anomaly. At the linear level this action is associated with the F-component of the chiral superfield

$$\Gamma^{dWG}(\phi, W^2) = (\ln\phi\, W_{\alpha\beta\gamma} W^{\alpha\beta\gamma})_F + ... \tag{49}$$

Terms with ... involve important corrections, required for locally superconformal action. The superfield $\ln\phi\, W_{\alpha\beta\gamma} W^{\alpha\beta\gamma}$ seems to have a Weyl weight $w = 3$, except that the $\ln\phi$ does not have a uniform scaling weight $w = 0$, which leads to complication and modification of the scale, chiral and S-supersymmetry transformations. Nevertheless, the consistent exact non-linear expression for N=1 superconformal anomaly in the form (46)

$$\delta\Gamma^{dWG}(\phi, W^2) \tag{50}$$

was established in [29] and given in eq. (5.7) there. It has terms with all local parameters in (47), i. e. there is a Weyl local conformal symmetry anomaly, local chiral U(1) symmetry anomaly, local S-supersymmetry anomaly and local Q-supersymmetry anomaly, all proportional to each other: either all of them or none. Note that the analysis in [29] was not based on specific computations of anomalies, it was an analysis based on consistency of the anomalies in N=1 superconformal models. The candidate consistent anomaly (46) is available, the coefficient X in (45) is model-dependent.

It may be useful also to bring up here the relevant discussion of the N=1 superconformal anomaly in [30, 31]. The corresponding gauge-independent[5] part of the anomaly is given by

$$\Gamma^{ST}(\phi, W^2) = 2(c-a) \int d^8 z \frac{E^{-1}}{R}\, \ln\phi\, W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} + c.c. \tag{51}$$

[5] We omit terms in [30] proportional to R and $G_{\alpha\dot\beta}$ superfields as they depend on the gauge-fixing condition and may be removed, as shown for example in [19].

and

$$\delta\Gamma^{ST}(\phi, W^2) = 2(c-a) \int d^8 z \frac{E^{-1}}{R}\, \delta\Sigma\, W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} + c.c., \tag{52}$$

where under the superconformal transformations the compensator superfield transforms as

$$\phi(x,\theta) \to e^{\,\Sigma(x,\theta)}\, \phi(x,\theta) \tag{53}$$

Therefore, when its vev is non-vanishing, one may try to define the Goldstone superfield, which according to [30] is 'dimensionless' and transforms by a superfield shift

$$\delta \ln\phi(x,\theta) \to \delta\Sigma(x,\theta) \tag{54}$$

and therefore

$$\delta \ln\phi(x,\theta) W^2(x,\theta) = \delta\Sigma(x,\theta) W^2(x,\theta) \tag{55}$$

Therefore, the local dilatations of the supermultiplet $\ln\phi$ are different from those of a multiplet with a particular Weyl weight. For example, for the chiral multiplet ($\phi = \{Z, \chi, F\}$) of the Weyl weight w the superconformal transformations, given for example in eq. (16.33) in [2]) have some w-dependent terms like

$$\delta Z = w(\lambda_D + i\lambda_T) Z + ... \tag{56}$$

where $\lambda_D(x)$ is a local dilatation and $\lambda_T(x)$ is a local chiral transformation. The same for χ, F, there are terms depending on w. These w-dependent terms are replaced by different transformations when the fields do not scale homogeneously under local dilatations. These transformations for $\ln\phi$ can be inferred from (54) where

$$\Sigma = \left\{\lambda_D + i\lambda_T, \sqrt{2}\eta, 0\right\} \tag{57}$$

and corresponding changes in the superconformal derivatives. The standard superconformal action for the multiplet, given for example in eq. (16.33) in [2]) is not superconformal invariant anymore due to these corrections to the superconformal transformations. However, its superconformal variation does not depend on the compensator. As a result, the complete non-linear expression for anomaly in eq. (5.7) in [29] is different from $\delta\Gamma^{ST}(\phi, W^2)$ in (52). The expression for the anomaly in (52) depends on manifestly Q-supersymmetric superfields and gives the impression that only Weyl, chiral and S-supersymmetry anomalies are consistent. Meanwhile, the extra terms in $\delta\Gamma^{dWG}(\phi, W^2)$ involve also the Q-supersymmetry anomaly, and therefore the complete non-linear expression for N=1 superconformal anomaly is not given in terms of superfields with manifest Q-supersymmetry, but in eq. (5.7) in [29].

Thus, a complete expression in eq. (5.7) in [29] for the superconformal anomaly $\delta\Gamma^{dWG}(\phi, W^2)$ of N=1 superconformal models contains local scale, chiral, S-supersymmetry and Q-supersymmetry anomaly. It is

generated by the superconformal variation of the expression $\Gamma^{dWG}(\phi, W^2)$. This is a construction of a consistent anomaly which we intend to generalize to the N=4 case.

N=4 case

For the N=4 superconformal anomaly the actions of the type (49), (51) are not available. The reason is the same as we have already explained with regard to candidate counterterms. In N=4 case the generalization of the N=1 case of arbitrary functions of a chiral compensator like $\mathcal{G}(\phi) = \phi^{-2n}$ are not available, such superfields can't be used to provide invariant actions. Indeed, the compensating multiplets are in this case the vector multiplets whose transformations close only using specific field equations. Therefore, one cannot manipulate with these multiplets, as we explained in the beginning of this sec. VI. This excludes also a possibility to use $\mathcal{G}(\phi) = \ln \phi$ for building superinvariants. Therefore there is no supersymmetric version of $\ln \phi(R - R^*)^2$ (for chiral anomaly). It is available in N=1 and N=2 superconformal theories but not available in N=4. The N=4 superconformally invariant version of $\ln(\phi^I_{ij}\eta_{IJ}\phi^{ijJ})(R - R^*)^2$ is not available.

The restrictions of N=4 superconformal symmetry are significantly stronger than the ones for N=1,2. Q-supersymmetry has a limited restriction on N-extended supergravity counterterms, and suggests that for N-extended supergravity the $L = N$ geometric on shell counterterms are available, the same prediction follows from N=1,2 superconformal models. However, for N=4, the symmetries allow only the local classical action and protect the model from anomalies and counterterms.

This supports our conjecture that N=4 superconformal models are quantum mechanically consistent and therefore we may trust the analysis of candidate counterterms based on N=4 superconformal symmetry, which predict the UV finiteness of perturbative theory.

B. Can we falsify our arguments using more general N=4 models?

1. Consider N=4 supergravity interacting with some number n of N=4 vector multiplets. The superconformal un-gauge-fixed version of this model is described in [5, 6]. It correspond to the model which we present in eq. (19) where η_{IJ} has six negative eigenvalues as well as n positive eigenvalues.

There is a 1-loop UV divergence in case of N=4 SG + N=4 vector multiplets, see for example [10]. There is also a corresponding counterterm in the underlying superconformal theory, it contains the square of the Weyl tensor. The linearized version of it [6] is given in eq. (3.17) of [4]. The complete non-linear action for the N=2 superconformal case is given in eq. (5.18) of [4].

The existence of this 1-loop counterterm is in agreement with N=4 supergravity analysis, as well as actual computations in [10]. In components is starts with $C^2_{\mu\nu\lambda\delta} + ...$ which corresponds to R^2 and $R^2_{\mu\nu}$ terms, as explained in eqs. (8), (9). These vanish for pure N=4 supergravity, corresponding to the model with 6 N=4 compensators, since only in pure supergravity $R = R_{\mu\nu} = 0$. In presence of matter multiplets, the counterterm has terms which do not vanish on shell, like $T^2_{\mu\nu}$ and T^2.

The one-loop N=4 square of the Weyl multiplet counterterm is superconformal by itself, it does not need N=4 compensators since it has a correct Weyl weight. This is why it escapes the problem with negative power of compensators, which is present for all N=4 superconformal invariants, starting with 3 loops. They need $\phi^{-2(L-1)}$ corresponding to $\kappa^{2(L-1)}$. Clearly, for L=1 there is no such dependence on a compensator.

2. Now we apply our method to N=4 conformal supergravity interacting with some N=4 vector multiplets [35]. This model is believed to be renormalizable but has ghosts. The superconformal counterterm corresponding to the square of the Weyl tensor is not excluded and it is not vanishing. It is not proportional to the equations of motion of conformal supergravity interacting with any number of vector multiplets. Thus the renormalizable UV divergences, proportional to the part of conformal supergravity classical action are expected. And since again, this particular unique superinvariant has the proper Weyl weight, the action does not depend on compensators. Therefore this counterterm escapes the problem with negative power of compensators.

It is rather interesting to see how all facts known about these various N=4 models fall into place. Our conjecture therefore, is that new computations will continue to support the N=4 superconformal symmetry of the model underlying pure N=4 supergravity.

VII. DISCUSSION

We have discussed here the pure N=4 Poincaré supergravity, which is a gauge-fixed version of the corresponding N=4 superconformal theory, the details of which, including the action in eq. (3.16), are given by de Roo in [5]. Here we have explained shortly the important details of the gauge-fixing to N=4 Poincaré supergravity at the

[6] The complete non-linear bosonic action was recently derived in [34] by integrating over the N=4 Yang-Mills fields.

simple level of the bosonic part of the theory, as well as the role of the conformal compensators, six vector multiplets with the wrong sign kinetic terms. In particular, we have explicitly presented a triangular gauge for the local $U(1)$ symmetry in which the superconformal model [5] becomes a pure N=4 supergravity model [26].

We argued that the N=4 superconformal action in [5] is unique and that the symmetry does not admit higher derivative actions. The argument about the uniqueness of the N=4 superconformal model is based on the open gauge algebra of the $SU(2,2|4)$ superconformal symmetry[7], which requires the equations of motion for the fermion partner of the compensator. This allowed de Roo in [5] to reconstruct the action consistent with the open algebra. Our argument about the uniqueness of the N=4 superconformal theory is related to the absence of the higher derivative superconformal invariants. Such invariants require the presence of negative conformal weight superfields, constructed from conformal compensators, which can be used in building new superconformal invariants. Since in N=4 the only matter multiplets that are available to serve as conformal compensators are vector multiplets with the open algebra, they do not provide the negative weight superfields which will allow to make the N=4 superconformal generalization of $\int d^4x\,\sqrt{-g}\,\phi^{-4}C^4$ counterterms, where C is the Weyl tensor. Therefore the R^4 UV divergences are forbidden by the N=4 superconformal symmetry of the un-gauge-fixed theory, assuming that we have all tools available for such a construction.

We have presented in the Appendix the detailed discussion of the difficulties with the "bottom up order by order attempts" to construct the corresponding higher derivative N=4 supergravity invariants. One may try to start from the known on shell N=4 superspace [40] candidate counterterms [24, 39] and deform the classical supersymmetry to reach the agreement with an exact deformation of classical theory studied in the N=2 theory [23]. The problem is the absence of a clear guiding principle in the N=4 case. Therefore one can view the computation in [8] as an indication that such a deformation may be indeed impossible since we already have all tools available and they do not produce higher order genuine supersymmetric invariants.

We have analyzed the situation with N=4 local superconformal anomalies based on earlier detailed studies of consistent anomalies in N=1 superconformal theories in [29] and in [30, 31]. We argued that there is no generalization of local superconformal N=1 anomalies to the N=4 case, the reason being the same as for counterterms. The anomaly candidate requires to use the superfield $\ln\phi$, the logarithm of the compensator field, for constructing a consistent anomaly. But it is not possible in N=4, for the same reason as the negative powers of ϕ are not available as building blocks for superinvariants.

This observation provides the simplest possible explanation of the computation in [8] where R^4 UV divergence in N=4 L=3 supergravity was found to cancel. Note that if this is the true explanation, it would mean also that no other higher loop UV divergences are predicted by the N=4 superconformal theory. Therefore our conjecture is falsifiable, as soon as the UV properties of the 4-loop N=4 supergravity will be known, they will either confirm or invalidate our conjecture.

The conjectured superconformal symmetry of N=4 supergravity supports UV finiteness arguments for N $\geq$ 4 supergravities. For these models the UV finiteness argument is associated with the Noether-Gaillard-Zumino deformed duality current conservation [13] and with local supersymmetry deformed by the presence of the higher derivative superinvariant [23]. Both arguments require the existence of the Born-Infeld type deformation of extended supergravities [23, 38]. In the particular case of N=4 such a Born-Infeld type deformation is not possible according to our current best understanding of superconformal symmetry, which is a supporting argument for the UV finiteness of the N $>$ 4 models. If the N=8 Born-Infeld supergravity would be available, one would be able to derive the N=4 one by supersymmetry truncation, in conflict with superconformal symmetry.

If our conjecture that the local superconformal symmetry explains the 3-loop UV finiteness in N=4 is confirmed by the 4-loop case, it will give us a hint that the models with superconformal symmetry without any dimensionful parameters may serve as a basis for constructing a consistent quantum theory where M_{Pl} appears in the process of gauge-fixing spontaneously broken Weyl symmetry.

ACKNOWLEDGMENTS

We are grateful to Z. Bern, R. Roiban, M. Roček, I. Tyutin for useful discussions and to A. Tseyltin for a significant impact on the development of this investigation and to A. Linde and G. 't Hooft for the stimulating discussions of the potential role of the superconformal symmetry in physics. The work of S.F. is supported by the ERC Advanced Grant no. 226455, Supersymmetry, Quantum Gravity and Gauge Fields (SUPERFIELDS). The work of R.K. is supported by SITP and NSF grant PHY-0756174. The work of A.V.P. is supported in part by the FWO - Vlaanderen, Project No. G.0651.11, and in part by the European Science Foundation Holograv Network.

[7] Note that the algebra is closed on the Weyl multiplet, therefore all local symmetry transformations are fixed. Meanwhile the fact that the algebra is open on the N=4 Maxwell multiplet may be also related to the need to use an infinite number of auxiliary fields to close the algebra [36].

Appendix A: Consistent deformation of N=4 supergravity?

Why at present there is a problem with a consistent deformation of pure N=4 supergravity with the R^4 term and what has to be done to solve it?

Since N=4 pure supergravity is a gauge-fixed version of the superconformal N=4 model, one can think about a deformation of the theory to accommodate higher derivative actions either at the superconformal level or at the level of the superPoincaré gauge-fixed theory.

This appendix has a purpose to compare the model with N=4 superconformal symmetry with the one studied in N=2. Here we will discuss the situation with N=2 supergravity at the superPoincaré level, following [23]. In the N=2 case first the genuine superconformal N=2 higher derivative action was provided explicitly in [22] in the general case and in the simplest possible case corresponding to minimal pure N=2 supergravity in [23].

In N=2 we start with the superconformal action (13), where it is known how to produce superinvariants depending on S^{-2} and $\bar{S}^{-2}$ chiral compensators. This action produces the N=2 superconformal version of the bosonic term given in (12) $\phi^{-4}(C_{....})^4$. As we explained in Sec. VI, no such superconformal invariant is available in the N=4 case.

However, we may try to continue bottom up and start with the already gauge fixed superconformal N=4 model, i. e. with N=4 supergravity where at least the nonlinear on shell supersymmetric R^4 counterterm is available [24, 39] based on the on shell superspace construction [40]. In absence of genuine local supersymmetry and in absence of auxiliary fields we can start from the classical action of N=4 supergravity and deform it by the known counterterm

$$S_1^{def}(\varphi) = S_0(\varphi) + \lambda\, S_{ct}(\varphi) \tag{A1}$$

First we compute the variation of this action under undeformed local transformation

$$\delta_0 S_1^{def} = \frac{\delta S_0}{\delta\varphi}\delta_0\varphi + \lambda\frac{\delta S_{ct}}{\delta\varphi}\delta_0\phi = \lambda\frac{\delta S_{ct}}{\delta\varphi}\delta_0\phi \tag{A2}$$

The first term in (A2) vanishes for generic field configurations according to the definition of a local supersymmetry of the classical action.

$$\frac{\delta S_0}{\delta\varphi}\delta_0\phi = 0 \tag{A3}$$

The second term, the supersymmetry variation of the counterterm, vanishes only when the classical equations of motion are satisfied. Therefore the best we can say is that

$$\delta_0 S_1^{def} = \lambda\frac{\delta S_{ct}}{\delta\varphi}\delta_0\varphi = \lambda\frac{\delta S_0}{\delta\varphi}\delta X(\varphi) \tag{A4}$$

which generically is not zero[8]. In fact, the counterterm structure does not allow an unambiguous extraction of what the $\delta X(\varphi)$ is since the on shell superspace construction [40] solves the geometric Bianchi identities only under condition that

$$\frac{\delta S_0}{\delta\varphi} = 0 \tag{A5}$$

and therefore terms in the counterterms proportional to $\frac{\delta S_0}{\delta\varphi}$ are not unambiguously defined. Since also the expression $\frac{\delta S_0}{\delta\varphi}$ under local classical supersymmetry transforms via a linear combination of $\frac{\delta S_0}{\delta\varphi}$, none of these are directly available from the on shell counterterms.

However, our recently acquired knowledge of the situation with genuine N=2 superinvariants where auxiliary fields are eliminated, teaches us that we have to modify the symmetry transformations so that $\delta\varphi = \delta_0\varphi + \lambda\delta_1\varphi$. We need to make the following steps. Assume that we somehow succeed to generalize the known counterterm to the stage where we can find δX by performing the variation. We will call this generalization $\hat{S}_{ct}$. In such case we have

$$\frac{\delta\hat{S}_{ct}}{\delta\varphi}\delta_0\varphi = \frac{\delta S_0}{\delta\varphi}\delta X(\varphi). \tag{A6}$$

This reminds the situation described in Sec. VI in eq. (43) where the variation of the action under supersymmetry is explicitly proportional to left hand side of the Dirac equation: if $\gamma^\mu\partial_\mu\chi = 0$ the supersymmetry variation of the action vanishes, otherwise it is proportional to $\gamma^\mu\partial_\mu\chi$ and does not vanish.

Now we get

$$\delta S_1^{def} = \frac{\delta S_0}{\delta\varphi}(\delta_0\varphi + \lambda\delta_1\varphi) + \lambda\frac{\delta\hat{S}_{ct}}{\delta\varphi}(\delta X + \lambda\delta_1\phi) \tag{A7}$$

Terms linear in λ cancel if

$$\delta_1\varphi = \delta X(\varphi) \tag{A8}$$

If we find $\hat{S}_{ct}$ with computable $\delta X(\varphi)$ in the N=4 case, we have identified $\delta_1\varphi$. We are then left with non-vanishing

$$\lambda^2\frac{\delta\hat{S}_{ct}}{\delta\varphi}\delta_1\phi \tag{A9}$$

[8] In general it is not zero but one can try to argue that maybe it is actually zero, and in such case none of the problems described in this appendix actually materialize. This is why this issue was not given proper attention in the past. In [23] it was shown that the modification of the classical supersymmetry transformation is necessary, the contribution from various auxiliary fields does not cancel for example in the non-linear part of the gravitino supersymmetry transformation. In N=4 supergravity analogous terms must be present to provide a consistent truncation from higher supersymmetries to N=2.

To cancel this one we have to find a next term in the action

$$S_2^{def}(\varphi) = S_0(\varphi) + \lambda\, \hat{S}_{ct}(\varphi) + \lambda^2 S_2(\varphi) \qquad \text{(A10)}$$

Assume that we can find the function $S_2(\varphi)$ and the next order of deformation $\delta_2\varphi$ such that

$$\delta S_2^{def} = \lambda^2 \Big(\frac{\delta S_0}{\delta\varphi}\delta_2\varphi + \frac{\delta \hat{S}_{ct}}{\delta\varphi}\delta_1\phi + \frac{\delta S_2}{\delta\varphi}\delta_0\varphi \Big) = 0 \qquad \text{(A11)}$$

for generic configuration of ϕ. This is an extremely strong condition: to find $S_2(\varphi)$ and $\delta_2\varphi(\varphi)$ such that the second term in (A11) will be compensated by

$$\frac{\delta S_0}{\delta\varphi}\delta_2\varphi + \frac{\delta S_2}{\delta\varphi}\delta_0\varphi \qquad \text{(A12)}$$

Assume this problem at the λ^2 level was solved.

Now we have the analogous problem at the λ^3 order when we take into account that we have suppressed the term

$$\lambda^3 \Big(\frac{\delta \hat{S}_{ct}}{\delta\varphi}\delta_2\phi + \frac{\delta S_2}{\delta\varphi}\delta_1\varphi \Big) \qquad \text{(A13)}$$

We need to find S_3 and $\delta_3\phi$. Same for all higher order terms, we have to find new actions S_n and extra symmetries $\delta_n\varphi$.

In N=2 we have a closed form answer, the complete λ-independent local supersymmetry transformations and the complete action in eq. (13) which is linear in λ. Expanding around the classical solutions for auxiliary fields we reproduce a procedure analogous to one described here, since we can we extract the values of $S_2(\varphi)$ and $\delta_2\varphi$, ... , S_n and $\delta_n\phi$ for any n from the complete supersymmetric N=2 theory.

Meanwhile in N=4 the first step, $S_{ct} \Rightarrow \hat{S}_{ct}$, is not known, and the infinite amount of next steps is also not known to exist and *there is no guiding principle*. In a sense, step by step finding if this completion is possible is not much easier than computing the loop corrections.

[1] S. Ferrara, M. Kaku, P. K. Townsend and P. van Nieuwenhuizen, "Gauging the Graded Conformal Group with Unitary Internal Symmetries," Nucl. Phys. B **129**, 125 (1977).

[2] D.Z. Freedman and A. Van Proeyen, "Supergravity", Cambridge U.P., 2012.

[3] E. Bergshoeff, M. de Roo, J. W. van Holten, B. de Wit and A. Van Proeyen, "Extended Conformal Supergravity And Its Applications," in Superspace& supergravity, ed. S.W. Hawking and M. Roček (CUP, 1981) 237.

[4] E. Bergshoeff, M. de Roo and B. de Wit, "Extended Conformal Supergravity," Nucl. Phys. B **182**, 173 (1981).

[5] M. de Roo, "Matter Coupling in N=4 Supergravity," Nucl. Phys. B **255**, 515 (1985).

[6] M. de Roo and P. Wagemans, "Gauge Matter Coupling In N=4 Supergravity," Nucl. Phys. B **262**, 644 (1985).

[7] B. de Wit and S. Ferrara, "On Higher Order Invariants In Extended Supergravity," Phys. Lett. B **81**, 317 (1979).

[8] Z. Bern, S. Davies, T. Dennen and Y. -t. Huang, "Absence of Three-Loop Four-Point Divergences in N=4 Supergravity," arXiv:1202.3423 [hep-th].

[9] P. Tourkine and P. Vanhove, "A R^4 non-renormalisation theorem in N=4 supergravity," arXiv:1202.3692 [hep-th].

[10] M. Fischler, "Finiteness Calculations For O(4) Through O(8) Extended Supergravity And O(4) Supergravity Coupled To Selfdual O(4) Matter," Phys. Rev. D **20**, 396 (1979).

[11] N. Marcus, "Composite Anomalies In Supergravity," Phys. Lett. B **157**, 383 (1985); P. di Vecchia, S. Ferrara and L. Girardello, "Anomalies of hidden local chiral symmetries in sigma models and extended supergravities," Phys. Lett. B **151**, 199 (1985).

[12] R. Kallosh, "On Absence of 3-loop Divergence in N=4 Supergravity," Phys. Rev. D **85**, 081702 (2012) [arXiv:1202.4690 [hep-th]].

[13] R. Kallosh, "$E_{7(7)}$ Symmetry and Finiteness of N=8 Supergravity," JHEP **1203**, 083 (2012) [arXiv:1103.4115 [hep-th]].

[14] Z. Bern, J. J. Carrasco, D. Forde, H. Ita and H. Johansson, "Unexpected Cancellations in Gravity Theories," Phys. Rev. D **77**, 025010 (2008) [arXiv:0707.1035 [hep-th]].

[15] S. Deser, "Scale invariance and gravitational coupling," Annals Phys. **59**, 248 (1970); J. L. Anderson, "Scale invariance of the second kind and the brans-dicke scalar-tensor theory," Phys. Rev. D **3**, 1689 (1971); R. E. Kallosh, "On the renormalization problem of quantum gravity" Phys. Lett. B **55**, 321 (1975); S. Deser, M. T. Grisaru, P. van Nieuwenhuizen and C. C. Wu, "Scale Dependence and the Renormalization Problem of Quantum Gravity," Phys. Lett. B **58**, 355 (1975); T. Kugo and S. Uehara, "Improved superconformal gauge conditions in the $\mathcal{N}=1$ supergravity Yang-Mills matter system", Nucl. Phys. **B222**, 125 (1983); R. Kallosh, L. Kofman, A. D. Linde and A. Van Proeyen, "Superconformal symmetry, supergravity and cosmology," Class. Quant. Grav. **17** (2000) 4269 [Erratum-ibid. **21** (2004) 5017] [hep-th/0006179]; I. Bars, "Constraints on Interacting Scalars in 2T Field Theory and No Scale Models in 1T Field Theory," Phys. Rev. D **82**, 125025 (2010) [arXiv:1008.1540 [hep-th]]; S. Ferrara, R. Kallosh, A. Linde, A. Marrani and A. Van Proeyen, "Superconformal Symmetry, NMSSM, and Inflation," Phys. Rev. D **83**, 025008 (2011) [arXiv:1008.2942 [hep-th]].

[16] R. E. Kallosh, "The Renormalization In Nonabelian Gauge Theories," Nucl. Phys. B **78**, 293 (1974). P. van

Nieuwenhuizen and C. C. Wu, "On Integral Relations For Invariants Constructed From Three Riemann Tensors And Their Applications In Quantum Gravity," J. Math. Phys. **18**, 182 (1977).

[17] M. H. Goroff and A. Sagnotti, "The Ultraviolet Behavior Of Einstein Gravity," Nucl. Phys. B **266**, 709 (1986). A. E. M. van de Ven, "Two loop quantum gravity," Nucl. Phys. B **378**, 309 (1992).

[18] G. 't Hooft and M. J. G. Veltman, "One loop divergencies in the theory of gravitation," Annales Poincare Phys. Theor. A **20**, 69 (1974).

[19] R. E. Kallosh, O. V. Tarasov and I. V. Tyutin, "One Loop Finiteness Of Quantum Gravity Off Mass Shell," Nucl. Phys. B **137**, 145 (1978).

[20] S. Deser, J. H. Kay and K. S. Stelle, "Renormalizability Properties Of Supergravity," Phys. Rev. Lett. **38**, 527 (1977). S. Deser and J. H. Kay, "Three Loop Counterterms For Extended Supergravity," Phys. Lett. B **76**, 400 (1978); S. Ferrara and B. Zumino, "Structure of Conformal Supergravity," Nucl. Phys. B **134**, 301 (1978).

[21] M. de Roo, J.W. van Holten, B. de Wit and A. Van Proeyen, "Chiral superfields in $N = 2$ supergravity, Nucl. Phys. B173 (1980) 175.

[22] B. de Wit, S. Katmadas and M. van Zalk, "New supersymmetric higher-derivative couplings: Full N=2 superspace does not count!," JHEP **1101**, 007 (2011) [arXiv:1010.2150 [hep-th]].

[23] W. Chemissany, S. Ferrara, R. Kallosh and C. S. Shahbazi, "N=2 Supergravity Counterterms, Off and On Shell," arXiv:1208.4801 [hep-th].

[24] G. Bossard, P. S. Howe, K. S. Stelle and P. Vanhove, "The vanishing volume of D=4 superspace," Class. Quant. Grav. **28**, 215005 (2011) [arXiv:1105.6087 [hep-th]].

[25] Z. Bern, J. J. Carrasco, L. J. Dixon, H. Johansson, D. A. Kosower and R. Roiban, "Three-Loop Superfiniteness of N=8 Supergravity," Phys. Rev. Lett. **98**, 161303 (2007) [hep-th/0702112]. Z. Bern, J. J. M. Carrasco, L. J. Dixon, H. Johansson and R. Roiban, "Simplifying Multiloop Integrands and Ultraviolet Divergences of Gauge Theory and Gravity Amplitudes," arXiv:1201.5366 [hep-th]. Z. Bern, J. J. Carrasco, L. J. Dixon, H. Johansson and R. Roiban, "The Ultraviolet Behavior of N=8 Supergravity at Four Loops," Phys. Rev. Lett. **103**, 081301 (2009) [arXiv:0905.2326 [hep-th]].

[26] E. Cremmer, J. Scherk and S. Ferrara, "SU(4) Invariant Supergravity Theory," Phys. Lett. B **74**, 61 (1978).

[27] A. K. Das, "SO(4) Invariant Extended Supergravity," Phys. Rev. D **15**, 2805 (1977). E. Cremmer and J. Scherk, "Algebraic Simplifications in Supergravity Theories," Nucl. Phys. B **127**, 259 (1977).

[28] E. Cremmer and B. Julia, "The SO(8) Supergravity," Nucl. Phys. B **159**, 141 (1979).

[29] B. de Wit and M. T. Grisaru, "Compensating Fields And Anomalies," Edited by I. A. Batalin et al.: Quantum field theory and quantum statistics, Vol. 2, 1987.

[30] A. Schwimmer and S. Theisen, "Spontaneous Breaking of Conformal Invariance and Trace Anomaly Matching," Nucl. Phys. B **847**, 590 (2011) [arXiv:1011.0696 [hep-th]].

[31] I. L. Buchbinder and S. M. Kuzenko, "Nonlocal Action For Supertrace Anomalies In Superspace Of N=1 Supergravity," Phys. Lett. B **202**, 233 (1988).

[32] J. Wess and B. Zumino, "Consequences of anomalous Ward identities," Phys. Lett. B **37**, 95 (1971).

[33] R. E. Kallosh and I. V. Tyutin, "The Equivalence theorem and gauge invariance in renormalizable theories," Yad. Fiz. **17**, 190 (1973) [Sov. J. Nucl. Phys. **17**, 98 (1973)].

[34] I.L. Buchbinder, N.G. Pletnev and A.A. Tseytlin, " Induced N = 4 conformal supergravity ", to appear.

[35] E. S. Fradkin and A. A. Tseytlin, "Conformal Supergravity," Phys. Rept. **119**, 233 (1985).

[36] W. Siegel and M. Roček, "On Off-shell Supermultiplets," Phys. Lett. B **105**, 275 (1981).

[37] G. Bossard and H. Nicolai, "Counterterms vs. Dualities," JHEP **1108**, 074 (2011) [arXiv:1105.1273 [hep-th]].

[38] J. J. M. Carrasco, R. Kallosh and R. Roiban, "Covariant procedures for perturbative non-linear deformation of duality-invariant theories," Phys. Rev. D **85**, 025007 (2012) [arXiv:1108.4390 [hep-th]]. W. Chemissany, R. Kallosh and T. Ortín, "Born-Infeld with Higher Derivatives," Phys. Rev. D **85**, 046002 (2012) [arXiv:1112.0332 [hep-th]]. J. Broedel, J. J. M. Carrasco, S. Ferrara, R. Kallosh and R. Roiban, "N=2 Supersymmetry and U(1)-Duality," Phys. Rev. D **85**, 125036 (2012) [arXiv:1202.0014 [hep-th]]. S. M. Kuzenko, "Nonlinear self-duality in N = 2 supergravity," arXiv:1202.0126 [hep-th].
R. Kallosh and T. Ortín, "New $E_{7(7)}$ invariants and amplitudes," arXiv:1205.4437 [hep-th].

[39] R. E. Kallosh, "Counterterms in extended supergravities," Phys. Lett. B **99** (1981) 122; P. S. Howe and U. Lindstrom, "Higher Order Invariants In Extended Supergravity," Nucl. Phys. B **181**, 487 (1981). P. S. Howe, K. S. Stelle and P. K. Townsend, "Superactions," Nucl. Phys. B **191**, 445 (1981).

[40] L. Brink and P. S. Howe, "The $\mathcal{N} = 8$ Supergravity In Superspace," Phys. Lett. B **88**, 268 (1979). P. S. Howe, "Supergravity In Superspace," Nucl. Phys. B **199**, 309 (1982).